3,(H) - Math.
hyperbolic function
paper 1st, Group A
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27.04.2021

(vi) $\log \sin(\theta + i\phi)$ (1)

$$= \log\{\sin\theta \cos i\phi + \cos\theta \sin i\phi\}$$

$$= \log\{\sin\theta \cosh\phi + i\cos\theta \sinh\phi\}$$

This is the form $\log(\alpha + i\beta)$

$$= \frac{1}{2}\log\{(\sin\theta\cosh\phi)^2 + \cos^2\theta \sinh^2\phi\} + i\tan^{-1}\frac{\cos\theta\sinh\phi}{\sin\theta\cosh\phi}$$

$$= \frac{1}{2}\log\{\sin^2\theta(\cosh^2\phi) + (1-\sin^2\theta)\sinh^2\phi\} + i\tan^{-1}(\cot\theta\tanh\phi)$$

$$= \frac{1}{2}\log\{\sin^2\theta\cosh^2\phi + \sinh^2\phi - \sin^2\theta\sinh^2\phi\} + i\tan^{-1}(\cot\theta\tanh\phi).$$

$$= \frac{1}{2}\log\{\sin^2\theta(\cosh^2\phi - \sinh^2\phi) + \sinh^2\phi\} + i\tan^{-1}(\cot\theta\tanh\phi)$$

$$= \frac{1}{2}\log\{\sin^2\theta(1) + \sinh^2\phi\} + i\tan^{-1}(\cot\theta\tanh\phi)$$ Ans

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$ab = c$

$\therefore b = c/a$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cosh\phi \sin\theta = \cos\alpha$

$\therefore \cosh\phi = \dfrac{\cos\alpha}{\sin\theta}$

6(a) If $\sin(\theta + i\phi) = \cos\alpha + i\sin\alpha$

Prove that $\sin\alpha = \pm\cos^2\theta = \pm\sin$ (2)

Here $\sin\theta\cos i\phi + \cos\theta\sin i\phi = \cos\alpha + i\sin\alpha$

or $\sin\theta\cosh\phi + i\cos\theta\sinh\phi = \cos\alpha + i\sin\alpha$

Equating real and imaginary parts

$\cos\alpha = \sin\theta\cosh\phi$ — (1)

$\sin\alpha = \cos\theta\sinh\phi$ — (2)

or $\sin^2\alpha = \cos^2\theta\sinh^2\phi$

$= \cos^2\theta\{\cosh^2\phi - 1\}$

from (1)

or, $\sin^2\alpha = \cos^2\theta\left\{\dfrac{\cos^2\alpha}{\sin^2\theta} - 1\right\}$

or $\sin^2\alpha = \cos^2\theta\left(\dfrac{\cos^2\alpha - \sin^2\theta}{\sin^2\theta}\right)$

or $\sin^2\alpha\sin^2\theta = \cos^2\theta\{\cos^2\alpha - \sin^2\theta\}$

$= \cos^2\theta\{\cos^2\alpha - (1 - \cos^2\theta)\}$

$= \cos^2\theta\{\cos^2\alpha - 1 + \cos^2\theta\}$

or $\sin^2\alpha\sin^2\theta = \cos^2\theta\cos^2\alpha - \cos^2\theta + \cos^4\theta$

or $\cos^4\theta + \cos^2\theta(\cos^2\alpha) - \cos^2\theta - \sin^2\alpha\sin^2\theta$

or $\cos^4\theta + \cos^2\theta(\cos^2\alpha - 1) - \sin^2\alpha(1 - \cos^2\theta)$

or, $\cos^4\theta + \cos^2\theta(\cos^2\alpha - 1) - \sin^2\alpha + \sin^2\alpha\cdot\cos^2$

or $\cos^4\theta + \cos^2\theta(\cos^2\alpha - 1 + \sin^2\alpha) - \sin^2\alpha = 0$

or $\cos^4\theta + \cos^2\theta(1 - 1) = \sin^2\alpha$

or $\sin^2\alpha = \cos^4\theta + 0$

$\therefore \sin\alpha = \pm\cos^2\theta$ proved

or $\cos^2\theta = \pm\sin\alpha$

Again from (2)

$\sin^2\alpha = \cos^2\theta \sinh^2\phi$

or, $\sin^2\alpha = \pm\sin\alpha \sinh^2\phi$

or $\sin\alpha = \pm\sinh^2\phi$ proved

(ii) If $\cos(\theta + i\phi) = R(\cos\alpha + i\sin\alpha)$, prove that

$$\phi = \frac{1}{2}\log\frac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}$$

Sol. $\cos\theta\cos i\phi - \sin\theta\sin i\phi$

$= R\cos\alpha + iR\sin\alpha$

or $\cos\theta . \cosh\phi - i\sin\theta\sinh\phi$

$= R\cos\alpha + iR\sin\alpha$

Equating,

$\cos\theta\cosh\phi = R\cos\alpha$ (1)

$-\sin\theta\sinh\phi = R\sin\alpha$ (2)

$$\cosh\phi = \frac{R\cos\alpha}{\cos\theta} \quad (3)$$

$$\sinh\phi = -\frac{R\sin\alpha}{\sin\theta} \quad (4)$$

$$\therefore \frac{\cosh\phi}{\sinh\phi} = -\frac{\cancel{R}\cos\alpha}{\cos\theta} \times \frac{\sin\theta}{\cancel{R}\sin\alpha}$$

$$= -\frac{\cos\alpha\sin\theta}{\sin\alpha\cos\theta}$$

By comp & dividendo

$$\frac{\cosh\phi + \sinh\phi}{\cosh\phi - \sinh\phi} = \frac{\cos\alpha\sin\theta - \sin\alpha\cos\theta}{\cos\alpha\sin\theta + \sin\alpha\cos\theta}$$

(4)

or $\dfrac{e^{\phi}}{e^{-\phi}} = \dfrac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}$

or $e^{2\phi} = \dfrac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}$

$\log e^{2\phi} = \log \dfrac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}$

$2\phi \log e = \log \dfrac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}$

$\therefore \phi = \dfrac{1}{2} \log \dfrac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}$ [$\because \log e = 1$]

~~9. (i) If $\sin(A+iB) = x+iy$, prove that~~

~~$x^2 \quad + y^2$~~

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except at $z = 0$

Proved

P.G. Sem-II paper-VI, unit-I complex integration Dr. (S.) Singh 27-4-2021

(1)

Complex variable

Domain (Region) :→ A set of points in the Argand plane is said to be connected set if any two of its points can be joined by a continuous curve, all of whose points belong to S.

Contour :→ By contour, we mean a continuous chain of a finite number of regular arcs.

If the contour is closed and does not intersect itself then it is called closed contour.

Example Boundaries of circle, Δ^s and rectangle.

Cauchy's Theorem :→ (Remember)

If a function $f(z)$ is analytic and single valued inside and

22 शुक्र

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If $f(z)$ is an analytic function of z and if $f'(z)$ is continuous at each point within and on a closed contour C, then

$$\int_C f(z)\, dz = 0$$

where C is any closed contour contained in D

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			1	2	3
5	6	7	8	9	10
12	13	14	15	16	17
19	20	21	22	23	24
26	27	28	29	30	

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on a simple closed contour C, then (2)

$$\int_C f(z)\,dz = 0$$

where C is any closed contour contained in D.

Proof → In the proof of this theorem we will use of apply Green's theorem for a plane which states:

If $P(x,y)$, $Q(x,y)$, $\frac{\partial P}{\partial y}$, $\frac{\partial Q}{\partial x}$ are all continuous (without break) functions within a domain D and if C is any closed contour in D, then

$$\int_C (P\,dx + Q\,dy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

Now, $\int_C f(z)\,dz = \int_C (u\,dx - v\,dy) + i\int_C (v\,dx + u\,dy)$ ——— (1)

We have

$$f'(z) = u_x + iv_x = v_y - iu_y \quad \text{——— (2)}$$

[By Cauchy's-Riemann eq's]

Here $f'(z)$ is continuously differentiable
So from (2) u_x, u_y, v_x, v_y all exist and are continuous in D.
Thus from Green's theorem
From (1)

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14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

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25 सोम

$$\int_C f(z)\,dz = \iint_D \left(-\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy + i\iint_D \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dx\,dy$$

$$= \iint_D \left(\cancel{-\frac{\partial v}{\partial x}} + \cancel{\frac{\partial v}{\partial x}}\right) dx\,dy + i\iint_D \left(\cancel{\frac{\partial u}{\partial x}} - \cancel{\frac{\partial u}{\partial x}}\right) dx\,dy$$

[By Cauchy's Reimann equations]

26 मंगल

$$= 0$$ Proved

Cauchy's Integral Formula :